1、

1. $\oint_S \mathbb{E}\cdot da = \frac{1}{\varepsilon_0}\int_V \rho dv \Rightarrow 4\pi r^2 \mathbb{E}_r = \frac{q}{\varepsilon_0}\hat{r}$

$q'\mathbb{E} = \frac{qq'}{4\pi\varepsilon_0 r^2}\hat{r}$

2、線形媒質

$\mathbb{D} = \varepsilon\mathbb{E},\ \mathbb{H} = \frac{1}{\mu}\mathbb{B}$

$\nabla\cdot\mathbb{D} = \rho_f \Rightarrow \nabla\varepsilon\cdot\mathbb{E} + \varepsilon\nabla\mathbb{E} = \rho_f$

$\therefore\ \nabla\cdot\mathbb{E} = \frac{1}{\varepsilon}(\rho_f - \nabla\varepsilon\cdot\mathbb{E})$

$\nabla\mathbb{H} = \mathbb{J}_p + \frac{\partial\mathbb{D}}{\partial t}$

$\Rightarrow \nabla\frac{1}{\mu}\times\mathbb{B} + \frac{1}{\mu}\nabla\times\mathbb{B} = \mathbb{J}_p + \varepsilon\frac{\partial\mathbb{E}}{\partial t}$

$\nabla\times\mathbb{B} = \mu\mathbb{J}_p + \mu\varepsilon\frac{\partial E}{\partial t} + \frac{1}{\mu}(\nabla\mu\times\mathbb{B})$

2、

1、ME(iv) より $\nabla\times\mathbb{B} = \mu_0\mathbb{J} + \mu_0\varepsilon_0\frac{\partial\mathbb{E}}{\partial t}$

両辺の発散をとる。

$0 = \mu_0\nabla\cdot\mathbb{J} + \mu_0\varepsilon_0\frac{\partial}{\partial t}\underbrace{(\nabla\cdot\mathbb{E})}_{(=\frac{\rho}{\varepsilon_0})}$

$\therefore\ \frac{\partial\rho}{\partial t} + \nabla\cdot\mathbb{J} = 0$

2. $\mathbb{J}_f = \nabla\times\mathbb{H} - \frac{\partial\mathbb{D}}{\partial t}$ を用いて $\mathbb{E}\cdot\mathbb{J}_f$ を計算する。

$\Rightarrow \mathbb{E}\cdot\mathbb{J}_f = -\left(\mathbb{E}\cdot\frac{\partial\mathbb{D}}{\partial t} + \mathbb{H}\cdot\frac{\partial\mathbb{B}}{\partial t}\right) - \nabla(\mathbb{E}\times\mathbb{H})$

$\frac{dW}{dt} = -\underbrace{\int_V\left(\mathbb{E}\cdot\frac{\partial\mathbb{D}}{\partial t} + \mathbb{H}\frac{\partial\mathbb{B}}{\partial t}\right)}_{\text{電磁場のエネルギー}} - \underbrace{\oint_S(\mathbb{E}\times\mathbb{H})da}_{\text{ポインティングベクトル}}$

線形媒質の場合

$\mathbb{D} = \varepsilon\mathbb{E},\ \mathbb{H} = \frac{1}{\mu}\mathbb{B}$　単位体積当たりのエネルギーをuとする。

$\frac{du}{dt} = \frac{1}{2}\left[\varepsilon\frac{\partial}{\partial t}(\mathbb{E}\cdot\mathbb{E}) + \frac{1}{\mu}\frac{\partial}{\partial t}(\mathbb{B}\cdot\mathbb{B})\right] = \frac{1}{2}\frac{\partial}{\partial t}\left[\mathbb{E}\cdot\mathbb{D} + \mathbb{B}\cdot\mathbb{H}\right]$

$u = \frac{1}{2}(\mathbb{E}\cdot\mathbb{D} + \mathbb{B}\cdot\mathbb{H}) = \frac{1}{2}\left(\varepsilon\mathbb{E}^2 + \frac{1}{\mu}\mathbb{B}^2\right)$

3.

1. $\nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rf)$

$$\frac{\partial^2}{\partial r^2}(rf) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(rf)$$

$$\Rightarrow \tilde{f} = \frac{\tilde{f}_0}{r}e^{i(kr-\omega t)}$$

2. $\mathbb{E} = \begin{pmatrix} E_0 \\ 0 \\ 0 \end{pmatrix}\cos(kz-\omega t+\delta_1)$, $\mathbb{B} = \begin{pmatrix} 0 \\ B_0 \\ 0 \end{pmatrix}\cos(kz-\omega t+\delta_2)$

$\langle \mathbb{E}\times\mathbb{B}\rangle = \frac{1}{2}\begin{pmatrix} 0 \\ 0 \\ E_0B_0 \end{pmatrix}\cos(\delta_1-\delta_2)$

$\tilde{\mathbb{E}} = \begin{pmatrix} E_0 \\ 0 \\ 0 \end{pmatrix}e^{i(kz-\omega t)}$, $\tilde{\mathbb{B}} = \begin{pmatrix} 0 \\ B_0 \\ 0 \end{pmatrix}e^{i(kz-\omega t)}$

$$\tilde{\mathbb{E}}\times\tilde{\mathbb{B}} = \begin{pmatrix} 0 \\ 0 \\ E_0B_0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ E_0B_0e^{i(\delta_1-\delta_2)} \end{pmatrix}$$

4.

1. $\langle S\rangle = \frac{1}{2}c\varepsilon_0\mathbb{E}_0^2 = 1000$ $\quad E_0 = 870\,\mathrm{V/m}$ $\quad B_0 = E_0/c = 2.9\times10^{-6}\,\mathrm{T}$

放射圧 $1000/c = 3.3\times10^{-6}\,\mathrm{N/m^2}$

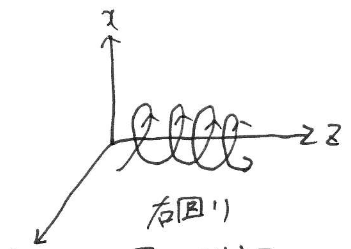
2. 右回り

$$\mathbb{E}_R = \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \end{pmatrix} = E_0\begin{pmatrix} e^{i(kz-\omega t)} \\ e^{i(kz-\omega t-\pi/2)} \end{pmatrix} = E_0e^{i(kz-\omega t)}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

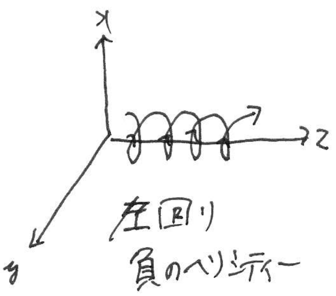
左回り

$$\mathbb{E}_L = E_0e^{i(kz-\omega t)}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\tilde{\mathbb{E}}_R+\tilde{\mathbb{E}}_L = 2E_0e^{i(kz-\omega t)}\begin{pmatrix} 1 \\ 0 \end{pmatrix},\quad \tilde{\mathbb{E}}_R+\tilde{\mathbb{E}}_L = -2E_0e^{i(kz-\omega t)}\begin{pmatrix} 0 \\ i \end{pmatrix}$$



右回り
正のヘリシティー



左回り
負のヘリシティー

5.

(1) $R=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$

$$\nabla\left(\frac{1}{R}\right)=-\frac{1}{R^2}\nabla R=-\frac{1}{R^2}\frac{\mathbb{R}}{R}=-\frac{\hat{\mathbb{R}}}{R^2}$$

(2) $$\nabla\rho=\begin{pmatrix}\frac{\partial t_r}{\partial x}\frac{\partial\rho}{\partial t_r}\\ \frac{\partial t_r}{\partial y}\frac{\partial\rho}{\partial t_r}\\ \frac{\partial t_r}{\partial z}\frac{\partial\rho}{\partial t_r}\end{pmatrix}=\frac{\partial\rho}{\partial t}\nabla t_r=\frac{\partial\rho}{\partial t}\left(-\frac{1}{c}\hat{\mathbb{R}}\right)$$

(3) $$\nabla\times\mathbb{J}=\begin{pmatrix}\frac{\partial t_r}{\partial y}\frac{\partial J_z}{\partial t_r}-\frac{\partial t_r}{\partial z}\frac{\partial J_y}{\partial t_r}\\ \text{〃}\end{pmatrix}=\begin{pmatrix}-\frac{1}{c}\frac{\partial R}{\partial y}\frac{\partial J_z}{\partial t}+\frac{1}{c}\frac{\partial R}{\partial z}\frac{\partial J_y}{\partial t}\\ \text{〃}\end{pmatrix}$$

$$=-\frac{1}{c}\nabla R\times\frac{\partial\mathbb{J}}{\partial t}=\frac{1}{c}\left(\frac{\partial\mathbb{J}}{\partial t}\times\hat{\mathbb{R}}\right)$$

(4) $\mathbb{E}=-\nabla V-\frac{\partial\mathbb{A}}{\partial t}$

$$\nabla V=\frac{1}{4\pi\varepsilon_0}\int_V\left[\nabla(\rho)\frac{1}{R}+\nabla\left(\frac{1}{R}\right)\rho\right]dv'=\frac{1}{4\pi\varepsilon_0}\int_V\left[-\frac{\dot\rho}{cR}\hat{\mathbb{R}}-\frac{\rho}{R^2}\hat{\mathbb{R}}\right]dv'$$

$$\frac{\partial\mathbb{A}}{\partial t}=\frac{\mu_0}{4\pi}\int_V\frac{\dot{\mathbb{J}}}{R}dv$$

$$\mathbb{E}=\frac{1}{4\pi\varepsilon_0}\int_V\left[\frac{\rho}{R^2}\hat{\mathbb{R}}+\frac{\dot\rho}{cR}\hat{\mathbb{R}}-\frac{\dot{\mathbb{J}}}{c^2R}\right]dv'$$

$\mathbb{B}=\nabla\times\mathbb{A}$

$$\mathbb{B}=\frac{\mu_0}{4\pi}\int_V\nabla\times\left(\frac{\mathbb{J}}{R}\right)dv'=\frac{\mu_0}{4\pi}\int_V\left[\frac{1}{R}(\nabla\times\mathbb{J})-\mathbb{J}\times\nabla\left(\frac{1}{R}\right)\right]dv'$$

$$=\frac{\mu_0}{4\pi}\int_V\left[\frac{\mathbb{J}}{R^2}\times\hat{\mathbb{R}}+\frac{\dot{\mathbb{J}}}{cR}\times\hat{\mathbb{R}}\right]dv'$$

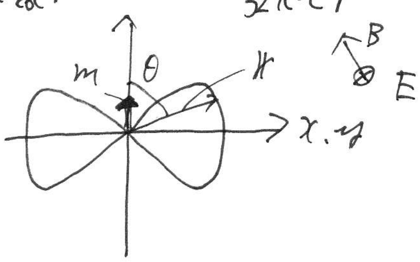
6.

1 (1) $\tilde{\mathbb{E}}=\frac{\mu_0 k\omega\tilde{m}_0}{4\pi r}\sin\theta\, e^{i(kr-\omega t)}\hat{\phi}$ $kr\gg1$, $\tilde{\mathbb{B}}=\frac{k^2\tilde{m}_0}{4\pi\varepsilon_0 c^2 r}\sin\theta\, e^{i(kr-\omega t)}\hat{\theta}$ $\lambda\ll r$

(2) $$\langle S\rangle=\frac{1}{2\mu}\mathrm{Re}(\tilde{\mathbb{E}}\times\tilde{\mathbb{B}}^*)=\frac{k^3\omega m_0^2}{32\pi^2\varepsilon_0 c^2 r^2}\sin^2\theta\,\hat{r}=\frac{\mu_0\omega^4 m_0^2}{32\pi^3 c^3 r^2}\sin^2\theta\,\hat{r}$$

全放射強度

$$P=\int\langle S\rangle da=\frac{\mu_0\omega^4 m_0^2}{12\pi c^3}$$



7 1.

(1) $X(x) = Ae^{ik_x x} + Be^{-ik_x x}$, $Y(y) = A'e^{ik_y y} + B'e^{-ik_y y}$

(2) $\begin{cases} x=0 \text{ で } \tilde{E}_z = 0 \text{ より } B = -A \\ x=a \text{ で } \quad " \quad \sin(k_x a) = 0 \quad k_x = \frac{m\pi}{a} (m = 0, 1, \cdots) \end{cases}$

$X(x) = 2iA\sin\left(\frac{m\pi}{a}x\right)$

yも同様

$Y(y) = 2iA'\sin\left(\frac{n\pi y}{b}\right) \qquad (n = 0, 1, \cdots)$

$\tilde{E}_z = E_a \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$

(3)

$K_{cmn} = \pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$, $\omega_{cmn} = c\pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$m = n = 1$, $a = 109\,mm$ $b = 55\,mm$ のとき $\omega = 19\,GHz$

8.

(1) $\sigma \ll \omega\varepsilon$ $\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left\{\left[1 + \frac{1}{2}\left(\frac{\sigma}{\varepsilon\omega}\right)^2 - 1\right]\right\} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$

$\sigma \gg \omega\varepsilon$ $\beta \simeq \omega\sqrt{\frac{\varepsilon\mu}{2}}\sqrt{\frac{\sigma}{\varepsilon\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}}$

$\beta \doteq \alpha = \frac{2\pi}{\lambda}$

(2) 水 $\frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}} \simeq 1 \times 10^{4}\,m$

金属 $\sqrt{\frac{2}{\omega\mu\sigma}} \simeq 1 \times 10^{-8}\,m$

9.

境界条件(ii)(iii)より $\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$

" (iv)より

$\frac{1}{\mu_1 v_1}(-\tilde{E}_{0I}\cos\theta_I + \tilde{E}_{0R}\cos\theta_R) = \frac{1}{\mu_2 v_2}(-E_{0T}\cos\theta_T)$

$\Rightarrow \tilde{E}_{0I} - \tilde{E}_{0R} = AB\tilde{E}_{0T}$

$\tilde{E}_{0R} = \left(\frac{1-AB}{1+AB}\right)\tilde{E}_{0T}$, $\tilde{E}_{0T} = \left(\frac{2}{1+AB}\right)\tilde{E}_{0I}$

$\mu_1 \simeq \mu_2$と近似

$r_s = \frac{n_1\cos\theta_I - n_2\cos\theta_T}{n_1\cos\theta_I + n_2\cos\theta_T} = -\frac{\sin(\theta_I - \theta_T)}{\sin(\theta_I + \theta_T)}$ フレネルの式

$t_s = \frac{2n_1\cos\theta_I}{n_1\cos\theta_I + n_2\cos\theta_T} = \frac{2\sin(\theta_T)\cos\theta_I}{\sin(\theta_I + \theta_T)}$

10-1

(1) $\theta_I=\theta_T=0$, $\mu_1\simeq\mu_2$ のとき

$A=\frac{n_2}{n_1}$, $B=1$ より

$$R=\left|\left(\frac{A-B}{A+B}\right)\right|^2=\left(\frac{n_1-n_2}{n_1+n_2}\right)^2,\quad T=AB\left|\left(\frac{2}{A+B}\right)\right|^2=\frac{n_2}{n_1}\left(\frac{2n_1}{n_1+n_2}\right)^2$$

$n_1=1.0$, $n_2=1.5$ のとき $R=4.0\times10^{-2}$, $T=9.6\times10^{-1}$

(2) $\theta_I=\frac{\pi}{2}\Rightarrow B=\frac{\cos\theta_T}{\cos\theta_I}\to\infty$, $R\to1$, $T\to0$

2.

$\varepsilon_1\simeq\varepsilon_0$, $\mu_1\simeq\mu_0$ より $n_1\simeq1.0$

$\tilde{A}=\tilde{n}_2=\frac{c}{\omega}(\alpha+i\beta)$

$\sigma\gg\varepsilon\omega$ のとき $\alpha\simeq\beta\simeq\sqrt{\frac{\omega\mu\sigma}{2}}=\frac{1}{d}$

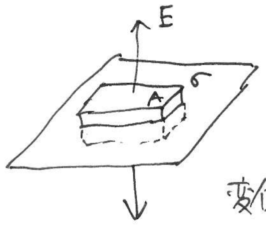
$$R=\left|\left(\frac{\tilde{A}-1}{\tilde{A}+1}\right)^2\right|=\left(\frac{\tilde{A}-1}{\tilde{A}+1}\right)\left(\frac{\tilde{A}^*-1}{\tilde{A}^*+1}\right)=\frac{1-\sqrt{\frac{2\omega\varepsilon}{\sigma}}+\frac{\omega\varepsilon}{\sigma}}{1+\sqrt{\frac{2\omega\varepsilon}{\sigma}}+\frac{\omega\varepsilon}{\sigma}}\simeq\frac{1-\sqrt{\frac{2\omega\varepsilon}{\sigma}}}{1+\sqrt{\frac{2\omega\varepsilon}{\sigma}}}$$

$$\simeq1-2\sqrt{\frac{2\omega\varepsilon}{\sigma}}=1-\frac{2\omega d}{c}$$

$T=1-R=\frac{2\omega d}{c}$　　σの大きな材料を選べばよい。

11

(1) 面電荷密度σの無限平面が作る電場



積分形のM.E.より　$\oint_S E\cdot da=\frac{1}{\varepsilon_0}\int_V\rho\,dV$

$\Leftrightarrow 2A|E|=\frac{\sigma A}{\varepsilon_0}$

$|E|=\frac{\sigma}{2\varepsilon_0}$

変位した電子イオン　$\sigma=\pm\frac{eNrA}{A}=\pm eNr$

$E=\frac{eNr}{\varepsilon_0}$

(2) $\frac{d^2r}{dt^2}=-\frac{e^2Nr}{\varepsilon_0 m_e}$　固有振動数 $\sqrt{\frac{e^2N}{\varepsilon_0 m_e}}$ の単振動

12. 太陽光が地球の大気を通る際に大気の分子によりレイリー散乱が起こる

$\sigma\simeq\frac{8}{3}\pi r_0^2\left(\frac{\omega}{\omega_0}\right)^6\propto\lambda^{-4}$　青い光が赤い光より散乱断面積が大きい。

朝、夕は昼にくらべて大気を通る距離が長くなり、青い光がより減衰することで赤くみえる。
厳密には太陽光のスペクトル(6000Kの黒体放射)
大気中のレイレー散乱・吸収、の感度で色が決まっている。